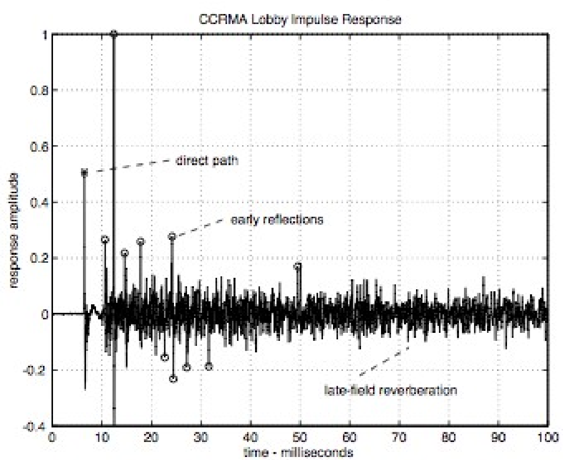
Impulse Response Measurements in the Presence of Clock Drift

Nicholas J. Bryan, Miriam A. Kolar, and Jonathan S. Abel

Stanford University | CCRMA

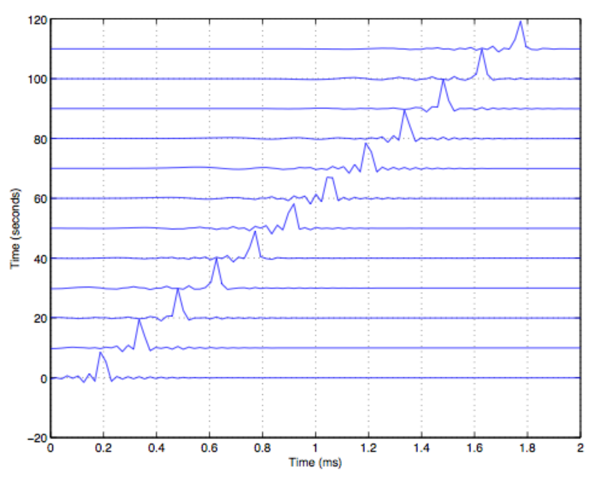
Introduction + Motivation

- Characterize the acoustic properties of a room in the form of an impulse response (IR)
- Occasionally difficult to record and playback on a single device as found in archeological acoustics (Chavín de Huántar—Miriam Kolar, et al.)

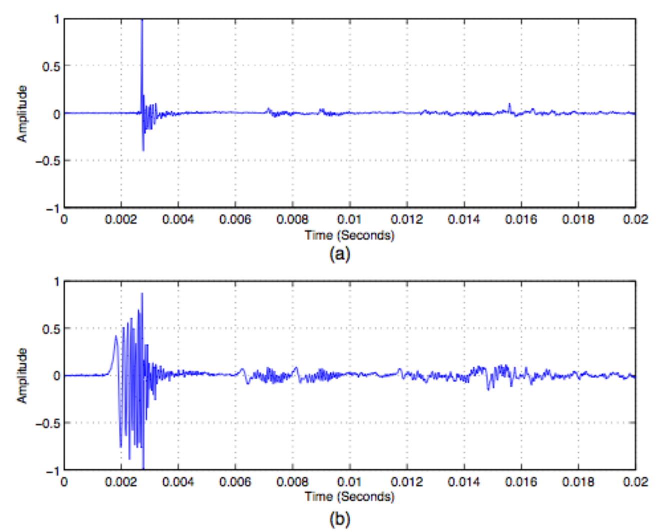


IR Measurements w/Clock Drift

- Different devices results in different digital clocks causing small timing differences in playback and record signals
- The misalignment accumulates over time for robust impulse response measurement techniques using convolution/correlation



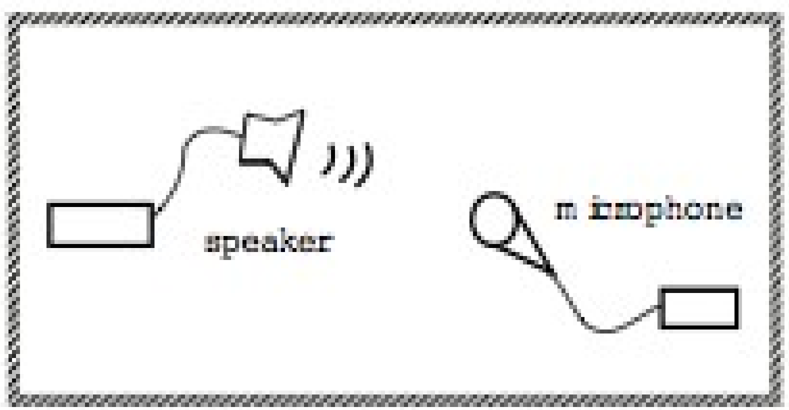
Recorded impulse train with clock drift over two minutes, drift 1-2 ms / minute



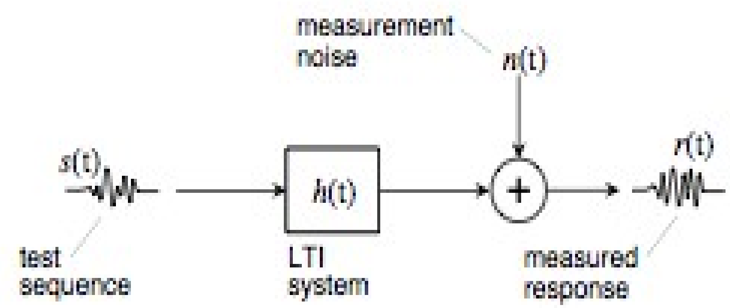
Measurement with and without clock drift

Impulse Response Measurements

- Measurement model $r(t) = s(t) * h(t) + n(t)$



Measurement system setup



Measurement model

Convolution IR Measurement Methods

Cyclic convolution

- Sinusoidal sweeps
  - linear, exponential, etc.
- Maximum length sequences
- Allpass chirps

Acyclic convolution

- **Sinusoidal sweeps**
  - linear, exponential, etc.
- Golay codes
  - binary, ternary, etc.

Sinusoidal Sweeps

- Increasingly popular, straightforward implementation
- Robust measurements against weak non-linearities
- Offer thorough theoretical analysis and alternative methods in the presence of clock drift

Acyclic IR Measurements

- Given the measurement model $r(t) = s(t) * h(t) + n(t)$ we assume, there exist an inverse filter $f(t)$ such that $s(t) * f(t) \approx \delta(t)$

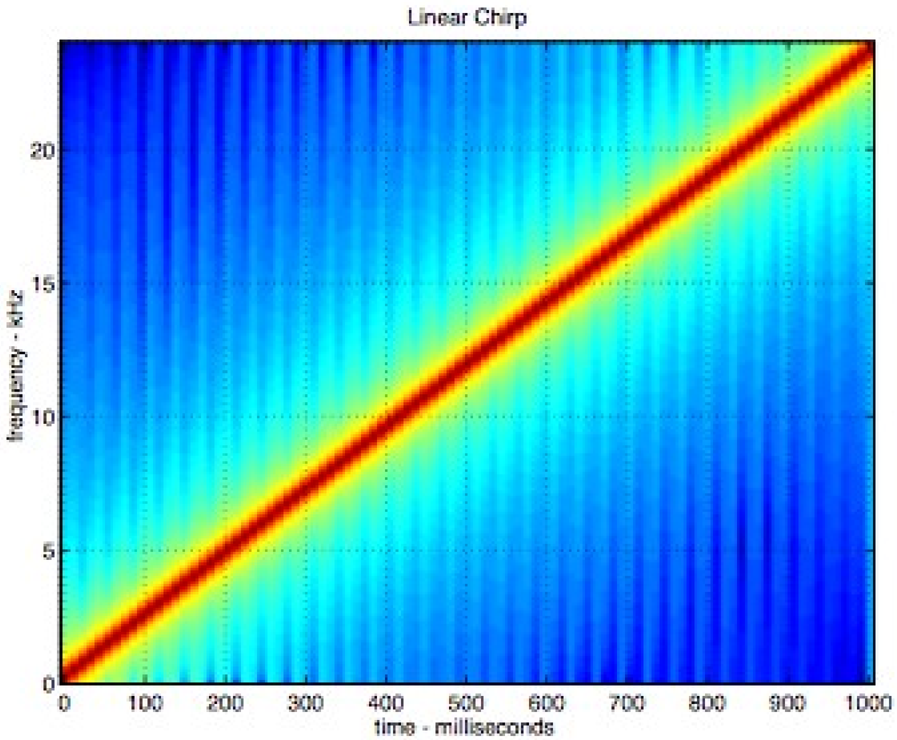
- The from the measurement model we can recover the impulse response via

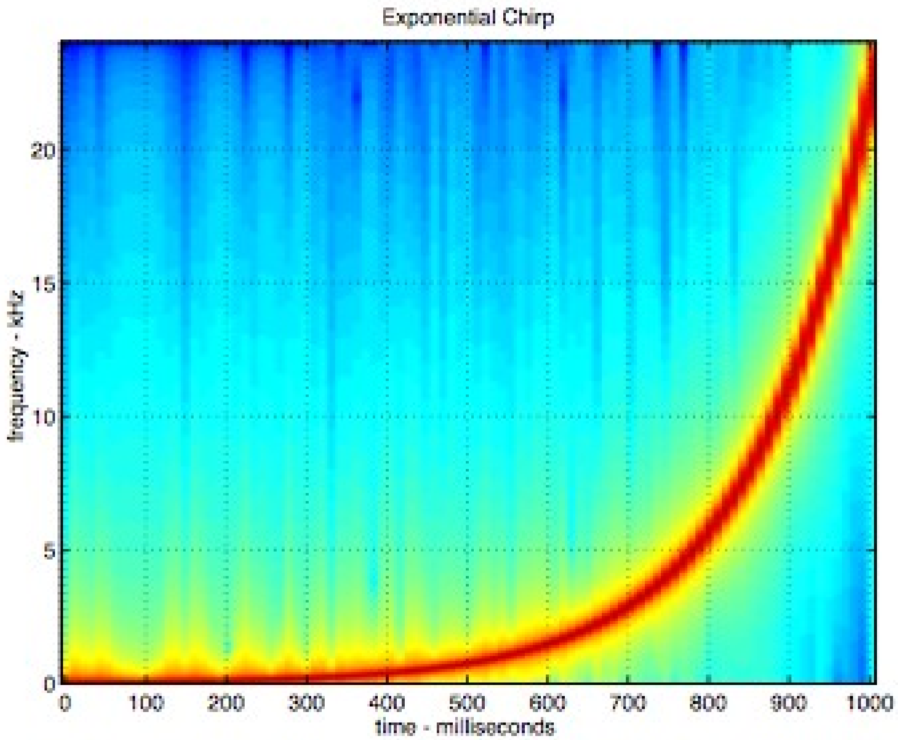
$$
\begin{aligned}
h(t) &= r(t) * f(t) \\
&= h(t) * s(t) * f(t) + n(t) * f(t) \\
&\approx h(t)
\end{aligned}
$$

Sinusoidal Sweeps

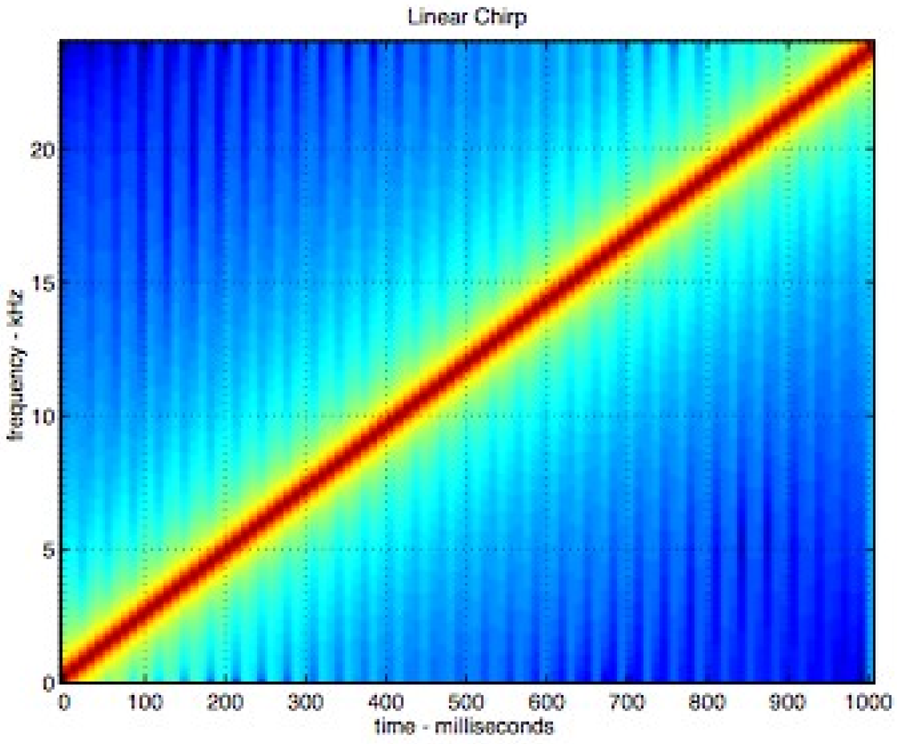
$$s(t) = \sin\phi(t) \qquad \phi(t) = \int_0^t \omega(\nu)d\nu$$

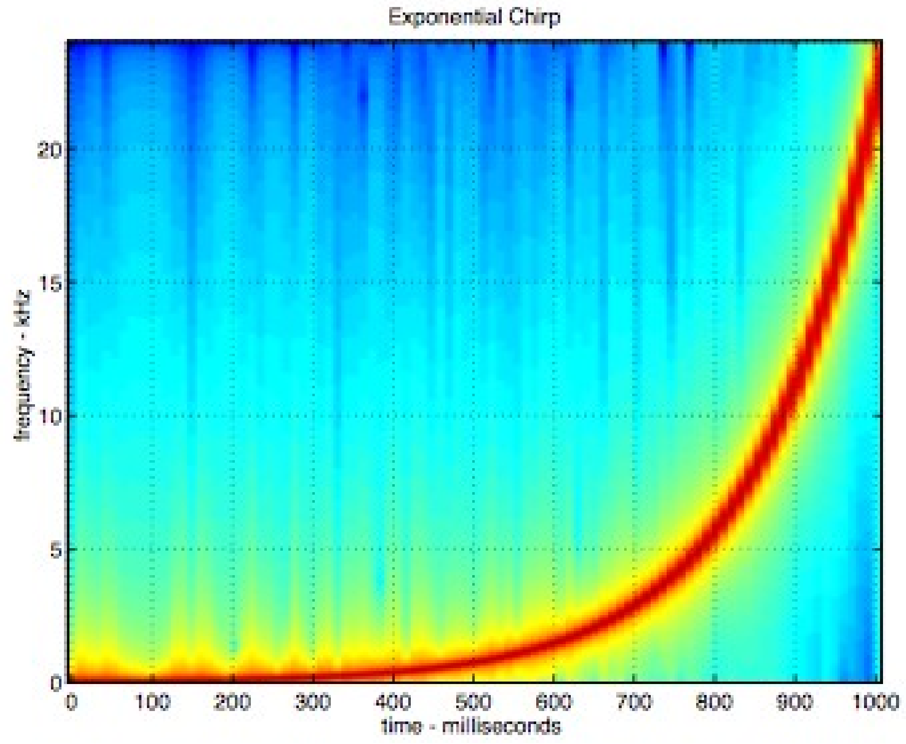
$$\omega_{lin}(t) = \left(\frac{\omega_1 - \omega_0}{\tau}\right)t + \omega_0 \qquad \omega_{exp}(t) = \omega_0 \exp\left\{ \frac{\mathrm{t}}{\tau} \ln \omega_1/\omega_0 \right\}$$





Sine Sweep Group Delay





- For smooth phase, the group delay is the time delay of the amplitude envelope of a sinusoid
- Functional inverse of frequency trajectory

Sinusoidal Sweep Transforms

- The frequency response $S(\omega)$ can be formulated via a magnitude and phase decomposition $|S(\omega)|e^{j\phi(w)}$

$$|S(\omega)| \approx 1 \Big/ \sqrt{\frac{1}{2}\left|\frac{d\gamma(\omega)}{d\omega}\right|} \qquad \phi_s(\omega) = -\int_0^{\omega} \gamma(\nu)d\nu$$

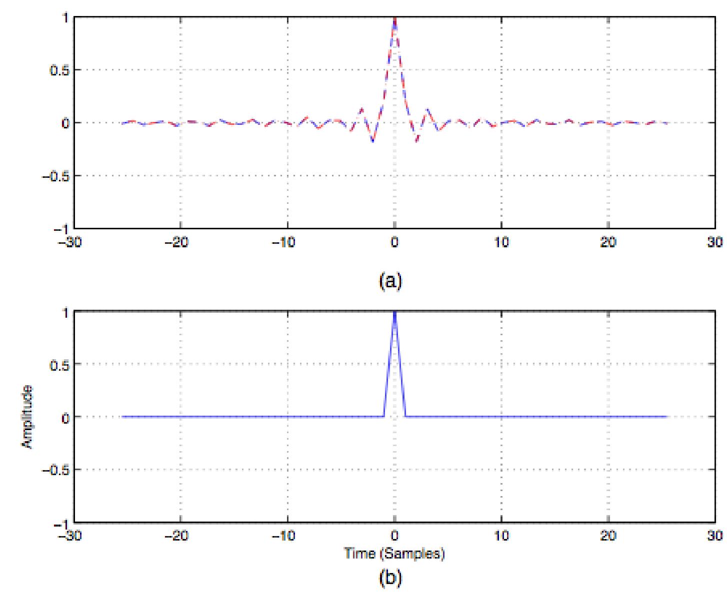
where the group delay $\gamma(\omega)$ is the functional inverse of $\omega(t)$ (Abel 2004)

Sinusoidal Sweep Inverse Filter

- $f(t)$ can be constructed using various methods
  - Time-reversal plus whitening filter
  - Numerical inversion via IFFT(1/FFT(s(t)))
  - **Closed form approximation**

$$f(t) \approx \left|\frac{d\omega(-t)}{dt}\right| s(-t)$$

$$|F(\omega)| \approx \sqrt{\frac{1}{2}\left|\frac{d\gamma(\omega)}{d\omega}\right|} \qquad \phi_f(\omega) = \int_0^{\omega} \gamma(\nu)d\nu$$



Clock Drift Analysis

- For convenience, a single clock can be chosen as reference with the other as drifting resulting in two scenarios
  - drifting playback clock
  - drifting record clock

$$\begin{aligned} h(t) &= r(t) * f(t) \\ &= h(t) * s(t) * f(t) \end{aligned}$$

IR deconvolution process is corrupted

Drifting Playback Clock

- Recorded signal $\tilde{r}(t) = h(t) * s(\alpha_p t)$
- Propagates through to the impulse response convolution process via

$$\begin{aligned}\tilde{h}(t) &= \tilde{r}(t) * f(t) \\ &= h(t) * (s(\alpha_p t) * f(t)) \\ &= h(t) * d_p(t)\end{aligned}$$

- IR is filtered by a drift filter

$$d_p(t) = s(\alpha_p t) * f(t)$$

Drifting Record Clock

- Recorded signal $\tilde{r}(t) = h(\alpha_r t) * s(\alpha_r t)$
- Propagates through to the impulse response convolution process via

$$\begin{aligned}\tilde{h}(t) &= \tilde{r}(t) * f(t) \\ &= h(\alpha_r t) * (s(\alpha_r t) * f(t))\end{aligned}$$

- IR is filtered by a drift filter $d_r(t) = s(\alpha_r t) * f(t)$ and resampled
- Resampling is typically negligible

Sine Sweep Clock Drift

- A stretch α in the time scale is equivalent to a stretch in the frequency trajectory and hence the group delay

$$\begin{aligned}\tilde{\gamma}(\omega) &\approx \alpha\gamma(\omega)\\ &\approx (1+\epsilon)\,\gamma(\omega)\end{aligned}$$

- As before, the magnitude and phase of a sweep can be solely expressed as a function of the group delay

Sine Sweep Transform w/Drift

- Modify the magnitude and phase of a sine sweep with drifting clocks resulting

$$|\tilde{S}(\omega)| \approx \sqrt{1+\epsilon}\,|S(\omega)| \qquad \tilde{\phi}_s(\omega) = -(1+\epsilon)\,\phi(\omega)$$

- The drift filter is then

$$\begin{aligned} D(\omega) &\approx \sqrt{1+\epsilon}\exp\{-\mathrm{j}\,\epsilon\,\phi_{\mathrm{s}}(\omega)\} \\ &\approx \exp\{-\mathrm{j}\,\epsilon\,\phi_{\mathrm{s}}(\omega)\} \end{aligned}$$

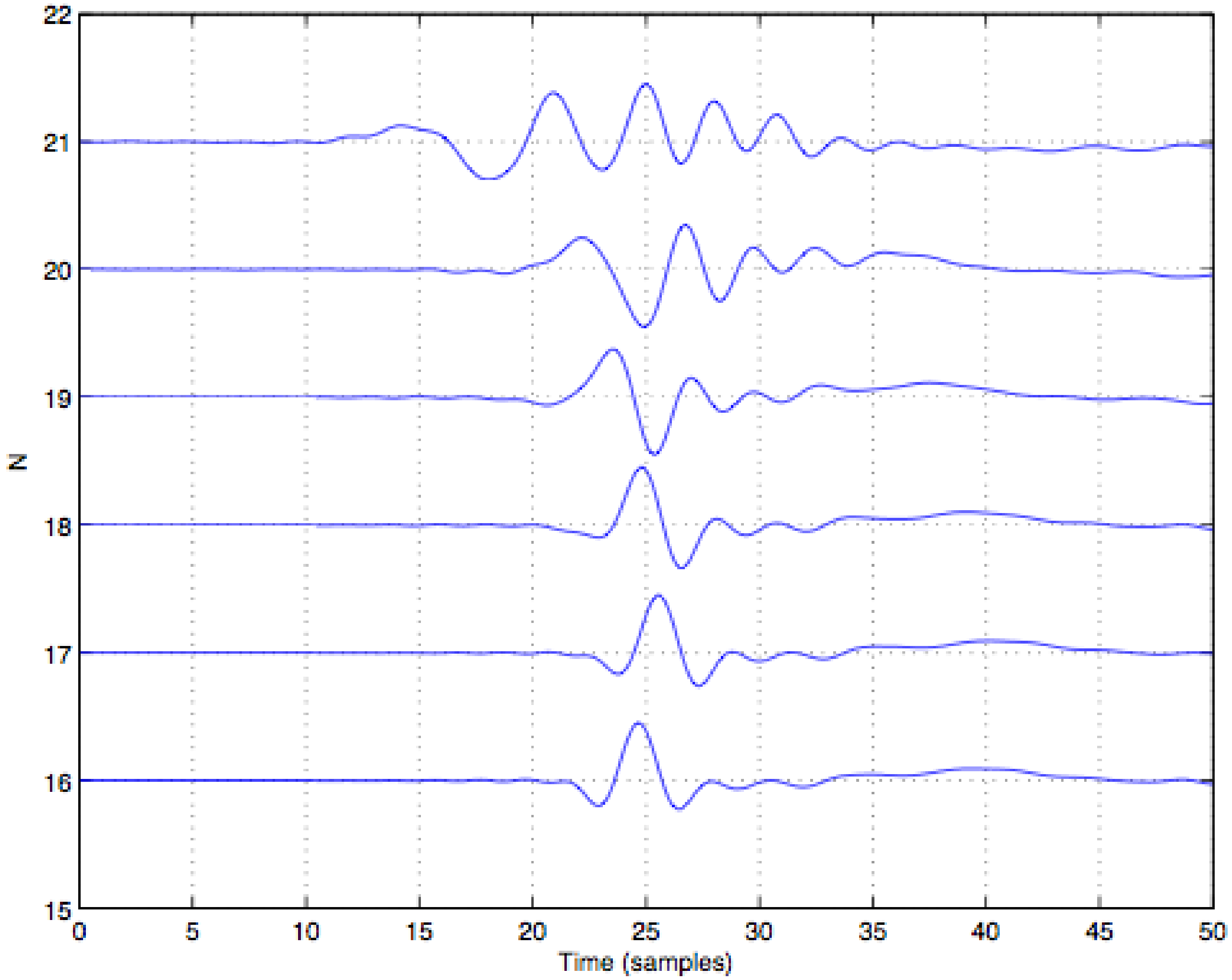
Sine Sweep Transform w/Drift

- The drift filter is an allpass filter for sine sweeps
  - Frequency trajectory of the same type as input sweep
  - Function of drift rate and sine sweep length

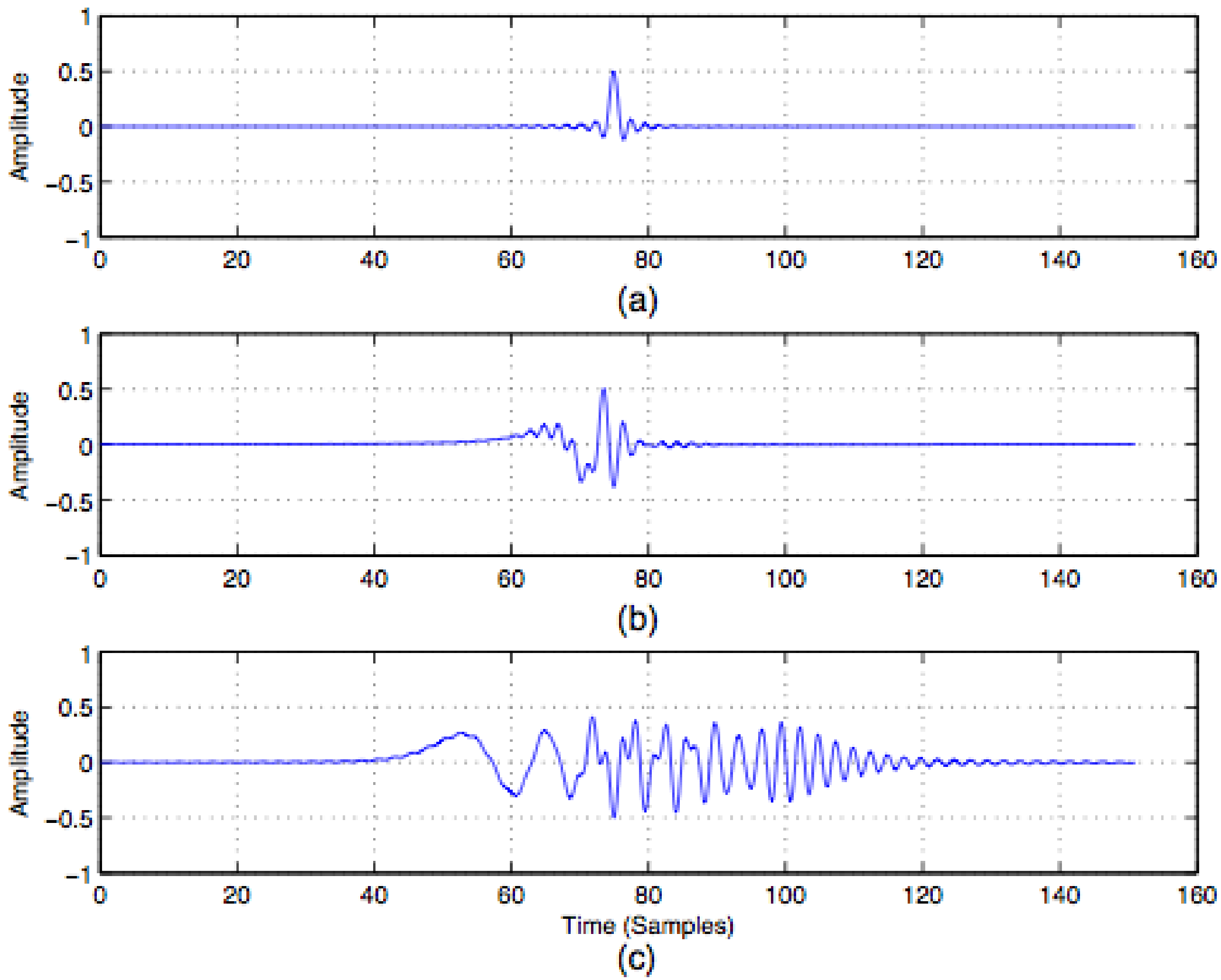
$$
\begin{aligned}
D(\omega) &\approx \sqrt{1+\epsilon}\,\exp\{-\mathrm{j}\,\epsilon\,\phi_{\mathrm{s}}(\omega)\} \\
&\approx \exp\{-\mathrm{j}\,\epsilon\,\phi_{\mathrm{s}}(\omega)\}
\end{aligned}
$$

Dependent on Sine Sweep Length

Sweep length in samples = 2^N



Dependent on Frequency Trajectory



No drift

Exponential trajectory w/drift

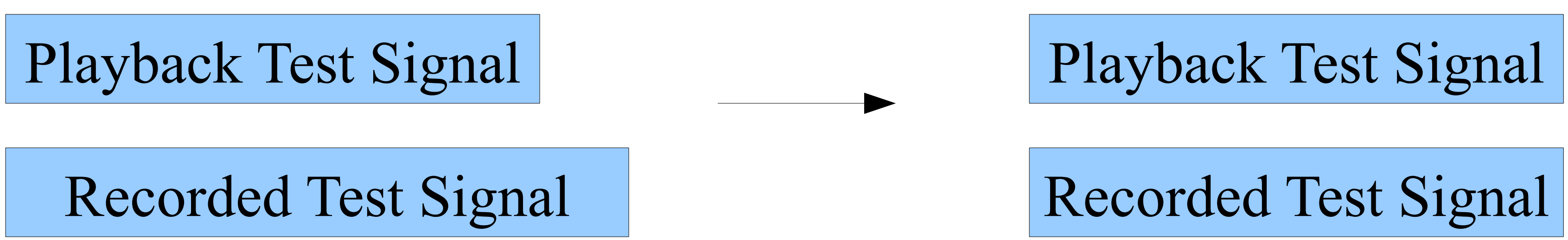
Linear trajectory w/drift

Clock Drift Compensation

- Desirable to remove drift effects via post-processing
  - Resampling
  - Compensation Filtering

Resampling Compensation

- Resample the recorded test signal or inverse filter prior to convolution
- Applicable to all convolution-based IR methods



Compensation Filtering

- Apply a compensation filter after convolution
- Applicable to sinusoidal sweeps
- Applied drift filter is allpass, so the compensation filter is the time-flip

Clock Drift Estimation

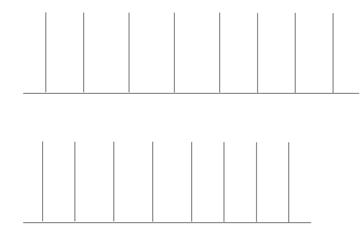
- Both compensation methods need an estimate of the clock drift
- Explicit Estimation
  - Direct electronic loopback recording of an periodic impulse train, noting the recorded time indices
- Implicit Estimation
  - Direct electronic loopback of sine sweep or inverse

Explicit Estimation

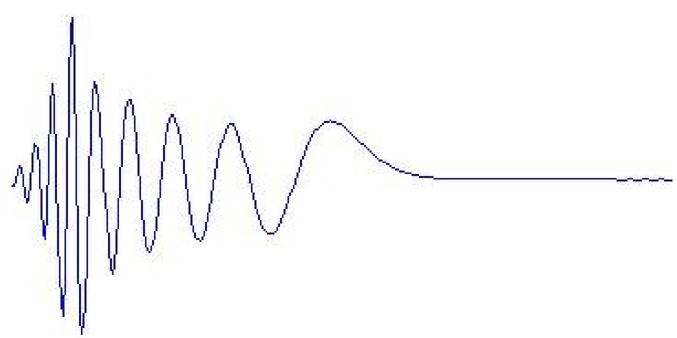
- Record high frequency impulse train

- Note time indices of peaks and compute least-squares estimate of the drift rate α via

$$\min_\alpha \| \mathbf{t} - \mathbf{t}_{meas}\alpha \|$$

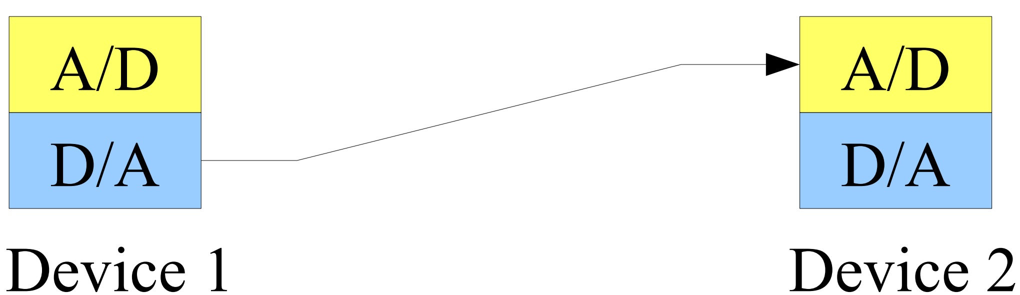


- Allpass chirps can be used for a more robust measure

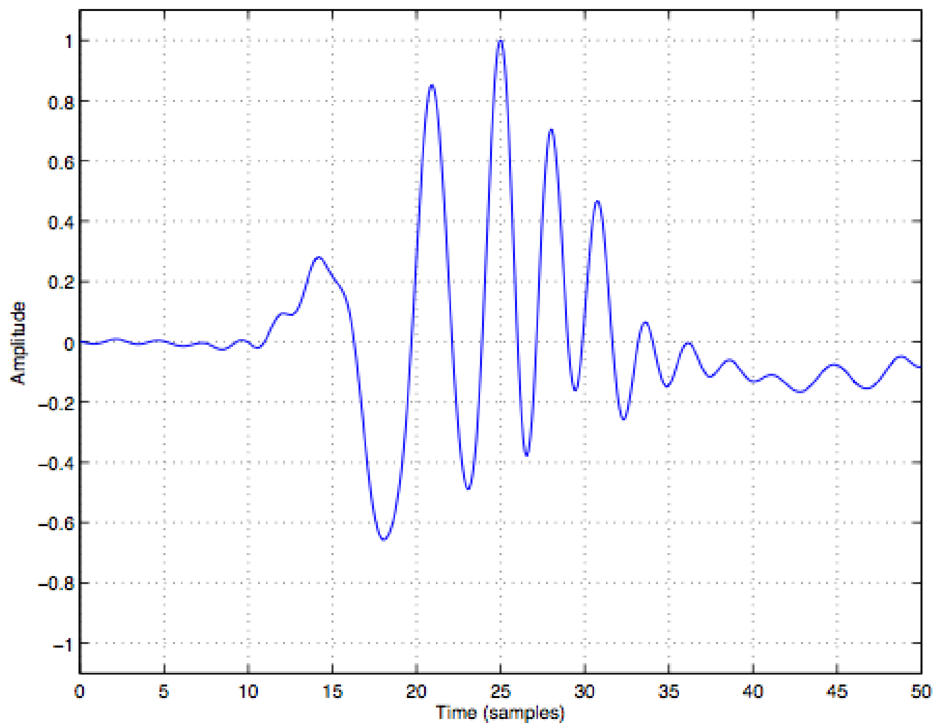


Implicit Estimation

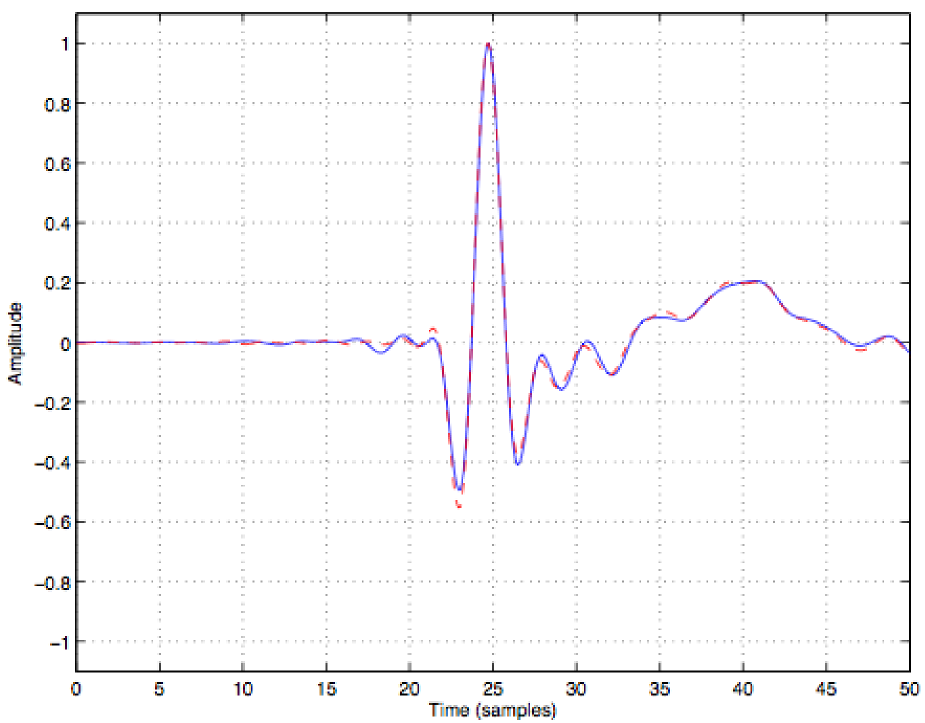
- Record direct loopback of sine sweep or inverse
- Simultaneously estimate drift and resample
- Drift rate *perfectly* estimated, but *unknown*



Compensation Results

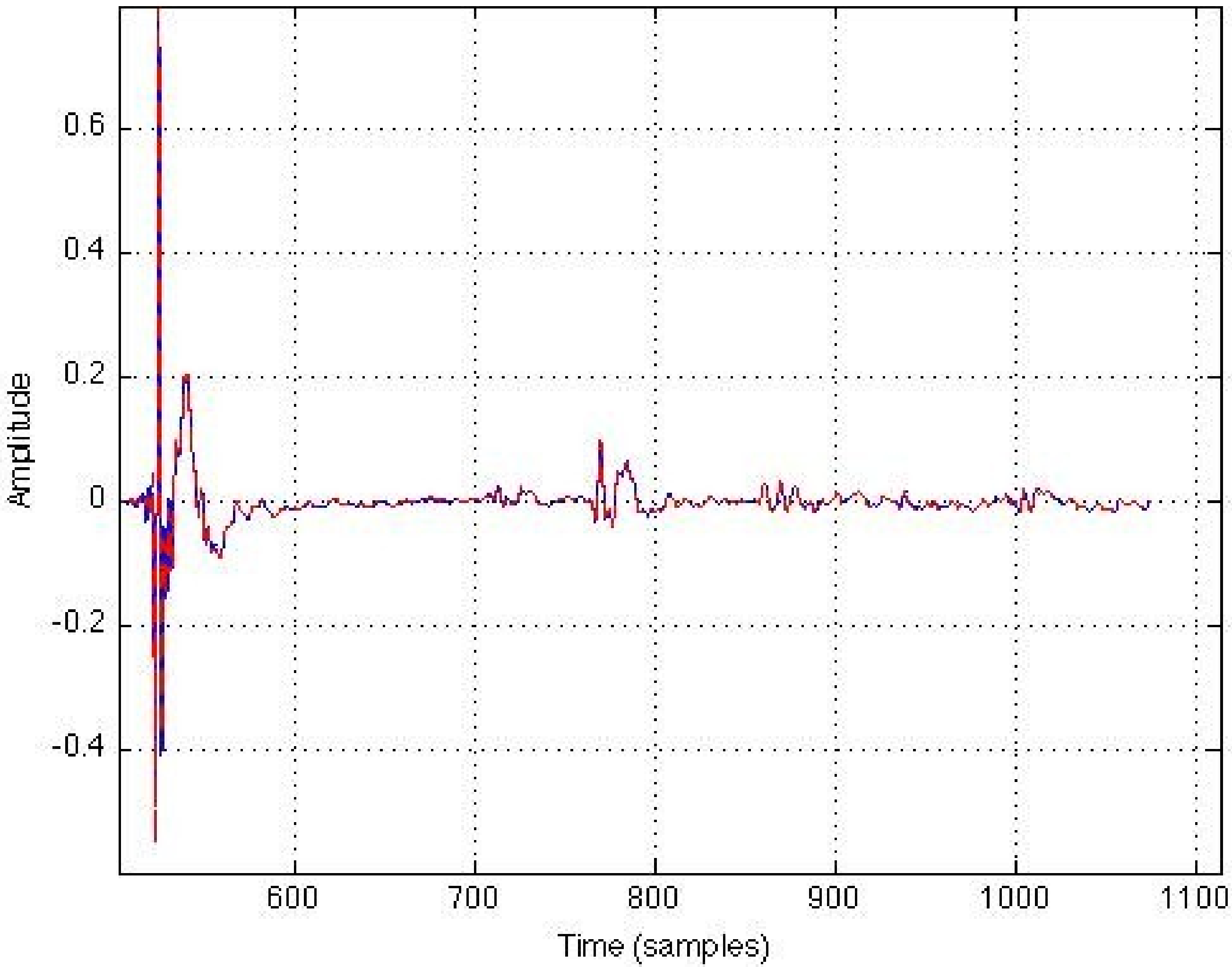


IR direct path with clock drift



IR compensated for drift vs. reference IR with no drift

Compensation Results



Compensated IR direct path and early reflections

Conclusions

- Clock drift can influence impulse response measurements in various ways
- Unwanted drift imposes allpass filtering on the resulting IR for sinusoidal sweep measurements
- Two methods of compensation are proposed and achieve near perfect compensation
- Post-processing solution for measuring room impulse responses in the presence of clock drift

Acknowledgements & Thank You!

- The Stanford Institute for Creativity and the Arts (SiCa), for the Chavín de Huántar Archaeological Acoustics Project

https://ccrma.stanford.edu/groups/chavin/